Evaluating Path Costs in Multi-Attributed Fuzzy Weighted Graphs

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Motivation

Suppose you had to plan a route to some goal, and were faced with multiple routes.

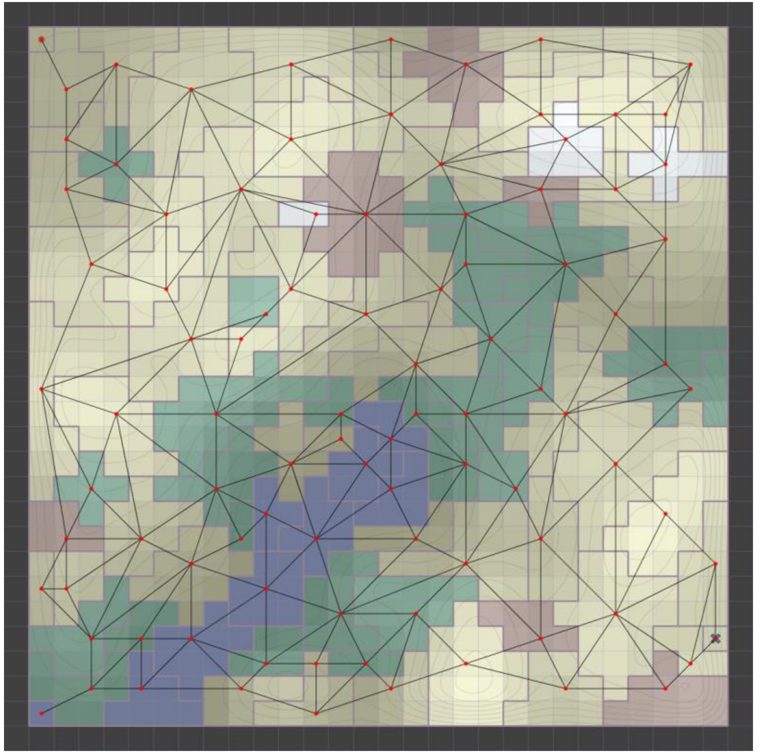
Each route has different qualities that make it more or less appealing.

Which route would you take?

How might you design an autonomous agent to act in your place?

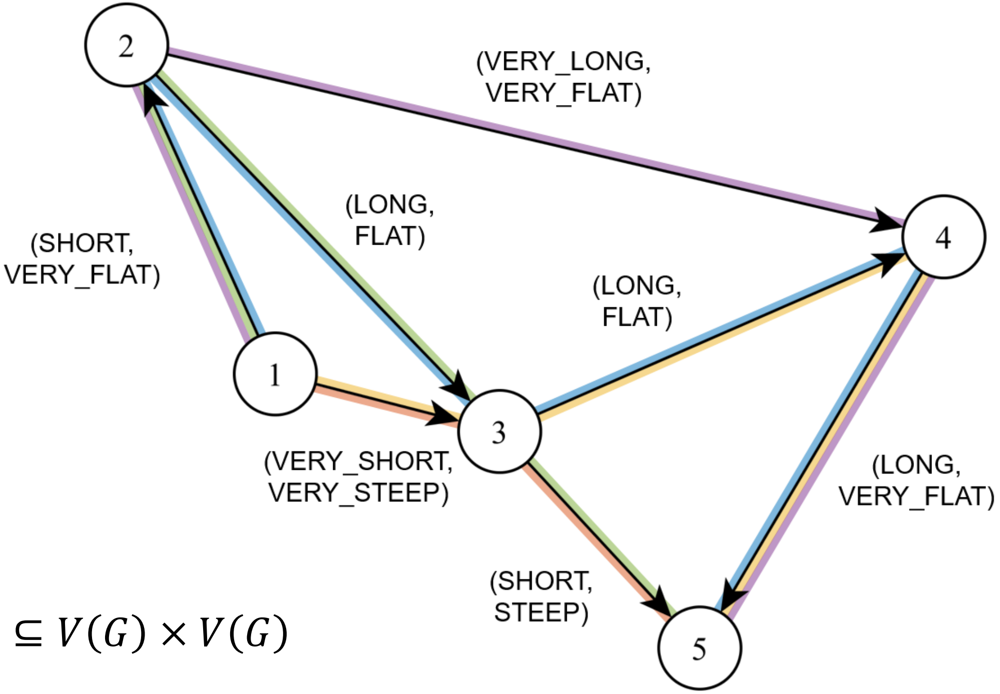
CMM Framework

- The Computational Mental Map (CMM) framework provides a simulation environment to experiment with different methods for multiobjective path planning.
- Procedurally generated grid world environments are used to create scenarios with multiple uncertain attributes.
- These are generic template problems that can be used as a way to study various aspects of the decision-making process.



Example Problem

- Represent the environment as a fuzzy weighted graph
- Five possible routes from 1 to 5
- Each path edge has a linguistic description of
  - Distance
  - Slope



Graph: G

Vertex: $v \in V(G)$

Edge: $e = (v_s, v_t) \in E(G)$, where $E(G) \subseteq V(G) \times V(G)$

Path: $p = (e_1, \dots, e_n) \in \left(E(G)\right)^n$

$P(s,t)$ is the set of all paths between vertices s and t

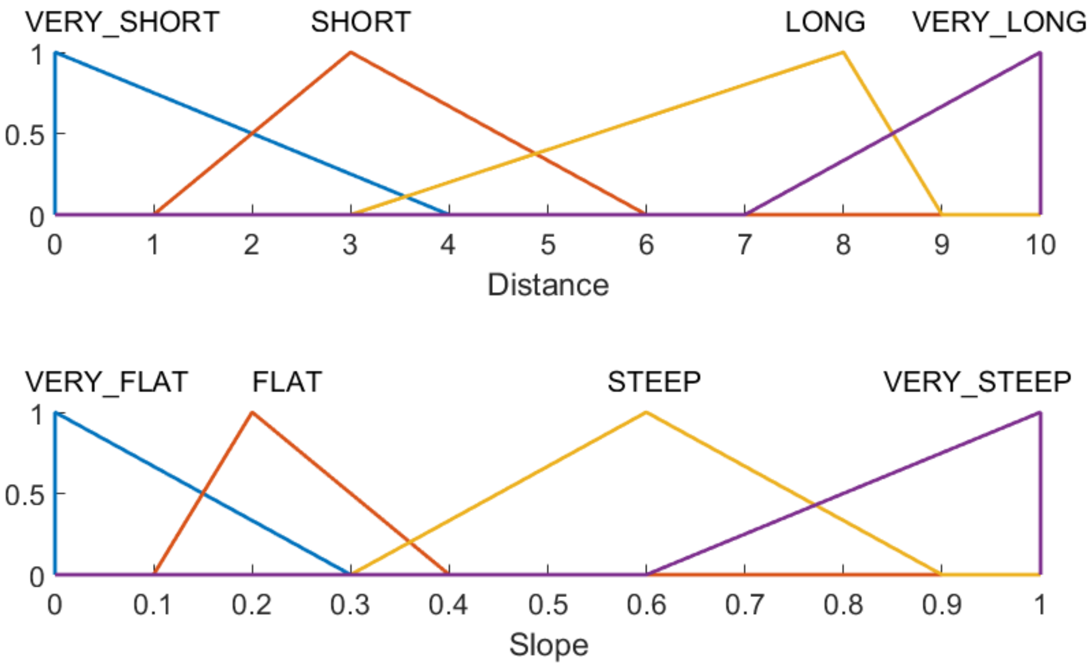
Fuzzy Numbers

- Each attribute is represented as a linguistic variable

- The variables may have different domains such that the values are not directly comparable
  - Distance in miles
  - Slope in percent

- Linguistic values are represented as triangular fuzzy numbers

$$A = \mathrm{Tri}(a, b, c)$$

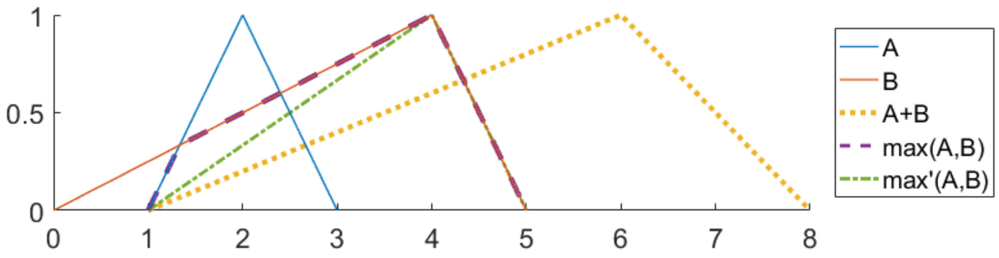


Fuzzy Operators

- A decision-maker needs to know the aggregate value of a feature over a path with multiple edges

- Use Zadeh's extension principle to define fuzzy operators

- We consider two operators:
  - Sum of distances
  - Maximum slope



$$A = \mathrm{Tri}(1,2,3) \qquad B = \mathrm{Tri}(0,4,5)$$
$$A + B = \mathrm{Tri}(1,6,8)$$
$$\max'(A,B) = \mathrm{Tri}(1,4,5)$$

$$\mathrm{Tri}(a_1, b_1, c_1) + \mathrm{Tri}(a_2, b_2, c_2) = \mathrm{Tri}(a_1 + a_2, b_1 + b_2, c_1 + c_2)$$
$$\max'\left(\mathrm{Tri}(a_1, b_1, c_1), \mathrm{Tri}(a_2, b_2, c_2)\right) = \mathrm{Tri}(\max(a_1, a_2), \max(b_1, b_2), \max(c_1, c_2))$$

Fuzzy Ranking

- For a least-cost path problem, the goal is to find a solution path that minimizes the objectives
- Fuzzy numbers capture uncertainty in the feature values, but there are many ways they can be ordered from smallest to largest
- One approach is to compute the weighted centroid:
  - For a triangular fuzzy number, the centroid is defined as $\bar{x} = \frac{1}{3}(a + b + c)$
  - The optimism/pessimism factor $\xi \in [0, 1]$ is used to interpolate between the minimum, centroid, and maximum values

$$C(A|\xi) = \begin{cases} a + 2\xi(\bar{x} - a), & \xi \leq 0.5 \\ \bar{x} + 2(\xi - 0.5)(c - \bar{x}), & \xi > 0.5 \end{cases}$$

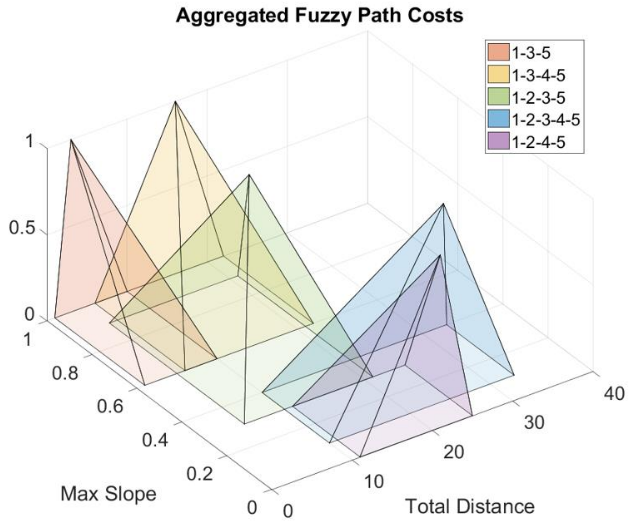
Aggregated Path Costs

- We compute the aggregated path cost as the total distance and maximum slope for each path

- Features: $\boldsymbol{F}(e) = \big(F_1(e), \ldots, F_m(e)\big)$
- Aggregated path cost: $\boldsymbol{A}(p) = \big(A_1(p), \ldots, A_m(p)\big)$
  - Summation: $A_i(p) = \sum_{j=1}^{n} F_i(e_j)$
  - Maximization: $A_i(p) = \max'_{j=1,\ldots,n} F_i(e_j)$

AGGREGATED FEATURE VALUES OF THE EXAMPLE GRAPH

Path	Color	Total Distance	Max Slope
1-3-5	Red	Tri(1, 3, 10)	Tri(0.6, 1, 1)
1-3-4-5	Yellow	Tri(6, 16, 22)	Tri(0.6, 1, 1)
1-2-3-5	Green	Tri(5, 14, 21)	Tri(0.3, 0.6, 0.9)
1-2-3-4-5	Blue	Tri(10, 27, 33)	Tri(0.1, 0.2, 0.4)
1-2-4-5	Purple	Tri(11, 21, 25)	Tri(0, 0, 0.3)



Multiobjective Optimization

- The Multiobjective Fuzzy Least Cost Path Problem (MO-FLCPP) is defined as

$$\begin{array}{ll} \text{minimize} & \boldsymbol{A}(p) = \big(A_1(p), \ldots, A_m(p)\big) \\ \text{subject to} & p \in P(s,t) \end{array}$$

- Typically, the objectives are conflicting such that they cannot all be minimized simultaneously.

- We say that a path p dominates a path p' $(p \prec p')$ iff if $A_i(p) \leq A_i(p')$ for all $i = 1, \ldots, m$ and there exists a $j \in \{1, \ldots, m\}$ such that $A_j(p) < A_j(p')$.

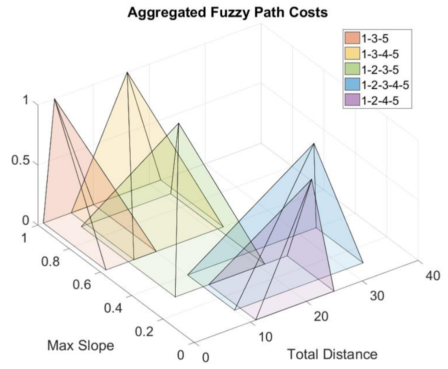
Pareto Optimality

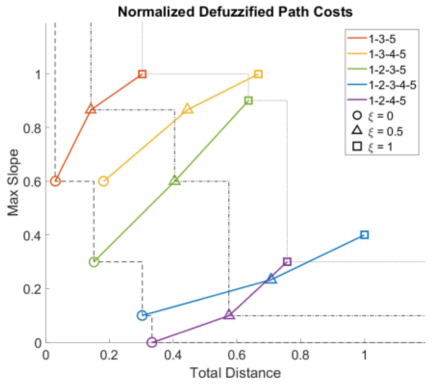
- If a path is not dominated by any other path, it is part of the Pareto optimal set

$$PS = \{p \in P(s,t) \mid \{p' \in P(s,t) \mid p' \prec p\} = \emptyset\}$$

- In the example problem, the yellow path is dominated by the red and green paths

- We can show how the paths compare by plotting the weighted centroid values with different optimism/pessimism weights.





Scalarization

- The aggregated cost vector of a path $\boldsymbol{A}(p)$ is a multidimensional vector of fuzzy numbers, where each component can have its own range.

- A normalized fuzzy vector $\boldsymbol{X}$ is obtained by scaling each dimension to an output range of $[0, 1]$.

- A scalarization function $g(\boldsymbol{X}|\boldsymbol{\lambda})$ reduces this multidimensional solution vector $\boldsymbol{X}$ to a real value using the weight vector $\boldsymbol{\lambda}$.

- $\boldsymbol{\lambda}$ represents the relative importance of each objective to the decision-maker, with higher weight given to more important objectives.

Choosing a Path

We consider three scalarization methods:

- Weighted sum

$$g^{\mathrm{ws}}(\boldsymbol{X}|\boldsymbol{\lambda}) = \sum_{i=1}^{m} \lambda_i X_i$$

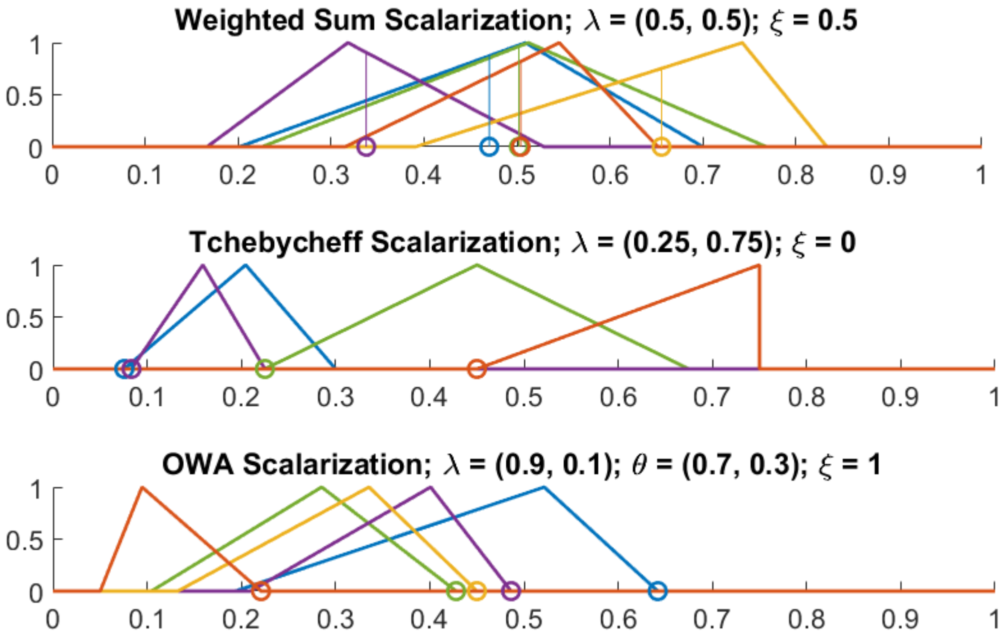
- Tchebycheff

$$g^{\mathrm{te}}(\boldsymbol{X}|\boldsymbol{\lambda}) = \max_{i=1,\ldots,m} \lambda_i X_i$$

- Ordered Weighted Average (OWA)

$$g^{\mathrm{OWA}}(\boldsymbol{X}|\boldsymbol{\lambda},\boldsymbol{\theta}) = \sum_{i=1}^{m} \theta_i B_{(i)}$$

where $B_{(i)}$ is the i^{th} largest $\lambda_i X_i$.
$\boldsymbol{\theta}$ is used to define different operators.



All Path Options

- The Tchebycheff and Weighted sum aggregations can be represented as OWA operators
  - Weighted sum: $\boldsymbol{\theta} = \left[\frac{1}{m}, \dots, \frac{1}{m}\right]$
  - Tchebycheff: $\boldsymbol{\theta} = [1, 0, \dots]$
- By changing $\boldsymbol{\lambda}$, ξ, and $\boldsymbol{\theta}$ we can make any Pareto optimal path have the lowest aggregated cost
- This adaptability of the model makes it suitable for learning a decision-maker's preferences from examples

BEST PATHS FOUND IN THE EXAMPLE GRAPH

		$\theta_1 =$	1	0.75	0.5	0.25	0
ξ	$\boldsymbol{\lambda}$	$\theta_2 =$	0	0.25	0.5	0.75	1
0	(0, 1)		P	P	P	P	–
	(0.25, 0.75)		B	P	P	P	P
	(0.5, 0.5)		G	P	P	P	P
	(0.75, 0.25)		G	G	R	R	P
	(1, 0)		R	R	R	R	–
0.5	(0, 1)		P	P	P	P	–
	(0.25, 0.75)		P	P	P	P	R
	(0.5, 0.5)		P	P	P	P	P
	(0.75, 0.25)		R	R	R	P	P
	(1, 0)		R	R	R	R	–
1	(0, 1)		P	P	P	P	–
	(0.25, 0.75)		P	P	P	P	R
	(0.5, 0.5)		P	P	P	P	P
	(0.75, 0.25)		R	R	R	P	P
	(1, 0)		R	R	R	R	–

Conclusions and Future Work

- Adding uncertainty with fuzzy numbers to a multiobjective least-cost path problem increases the ways that a decision-maker can choose a solution and allows the model to capture a wide range of agent behaviors.

- We did not discuss how to compute the paths for comparison
  - In large graphs, we cannot enumerate all paths
  - Instead, we can use a multiobjective evolutionary algorithm to find good paths
    - This approach is well-suited to nonlinear aggregation

- The CMM framework can generate many different types of problems for studying decision-making behavior
  - Problems like the traveling salesman with multiple objectives and partial observability