An α-level OWA Implementation of Bounded Rationality for Fuzzy Route Selection

Andrew R. Buck, James M. Keller, and Mihail Popescu

Department of Electrical and Computer Engineering
University of Missouri
Columbia, Missouri, USA
AndrewBuck@mail.missouri.edu
KellerJ@missouri.edu
PopescuM@missouri.edu

Abstract. As people move through uncertain environments, they are often presented with multiple route choices. Deciding which route to take requires an understanding of the environmental features and how they affect the person's interpreted cost of each route. These quantities can be appropriately modeled as fuzzy numbers to capture the inherent uncertainty in human knowledge. We present an approach to guide a person's decision-making process through an environment modeled as a fuzzy weighted graph, using an α-level OWA operator to implement the principle of bounded rationality. A cost value is computed for each possible route choice, which can then be used to rank the set of routes and make a decision.

1 Introduction

Human geography is a diverse field involving the study of human traits in geographic space. One aspect of human geography is the study of how people navigate through environments. In contrast to many computational path-planning algorithms, humans do not always make optimal decisions when moving in an environment. Rather, we make decisions based on a cognitive map built from spatial knowledge and experience [1]. Rarely do these maps contain perfect information, as locations and spatial relationships between objects are measured using humanistic concepts such as "There is a hill off in the distance," or "This path is about three miles long." This type of uncertainty can be modeled using fuzzy sets.

An environment can be viewed as a graph of discrete locations represented as vertices and path transitions represented as edges. A person, or agent, may assign a cost value to each path segment based on their personal interpretation of the environmental features of that segment and how those features affect their mobility along the path. The uncertainty inherent in the agent's perception is modeled by using fuzzy numbers to represent the costs. In order to evaluate the total cost of a route between two locations, an agent must aggregate the costs of each route segment. By studying decisions made by artificial agents in an agent-based modeling scheme, we can gain insight on how groups of people will move in their environment under stress conditions, one of the goals of human geography.

The focus of this paper is to present a method for determining the cost assigned to a particular route, based on environmental features and an agent's attributes. We use an α-level Ordered Weighted Average (OWA) operator [2] to implement bounded rationality [3], the idea that agents have limited resources with which to make decisions, resulting in sub-optimal choices. Once a route cost has been established in the form of a fuzzy number, a variety of path-planning algorithms can be used to guide the agent's decision-making process. These include standard fuzzy shortest path algorithms such as [4] and [5] or the genetic algorithm approach of [6]. The remainder of this paper is outlined as follows. In Sect. 2, we define the concepts of fuzzy numbers, fuzzy weighted graphs, and bounded rationality as implemented by an α-level OWA operator. In Sect. 3, we present an example scenario consisting of three different routes and show how different agent types evaluate the environment differently. Our conclusions and ideas for future work are given in Sect. 4.

2 Path Planning in Uncertain Environments

2.1 Fuzzy Numbers

A fuzzy number is a convex, normalized fuzzy set $A : \mathbb{R} \to [0, 1]$ that provides a way of representing uncertainty in the value of a real number. The membership function $\mu_A(x)$ gives the degree of membership that a specific value x has in the fuzzy number A. Using Zadeh's extension principle, we can define the arithmetic operators for fuzzy numbers, as well as other functions such as maximization and minimization. For a function $f(A, B)$ operating on two fuzzy numbers A and B, the resulting fuzzy number is given as

$$\mu_{f(A,B)}(z) = \sup_{z=f(x,y)} \min\big(\mu_A(x), \mu_B(y)\big). \tag{1}$$

Because fuzzy numbers are convex, we can use α-cuts and interval arithmetic to quickly compute the result of a fuzzy computation. An α-cut of a fuzzy number is an interval ${}^{\alpha}A = [l, r]$ such that $\mu_A(x) \geq \alpha, x \in [l, r]$. The decomposition theorem states that a fuzzy number is simply the union of all α-cuts, $\alpha \in [0, 1]$. For each value of α, the result of a convex, continuous function $f({}^{\alpha}A, {}^{\alpha}B)$ on the α-cuts of two fuzzy numbers ${}^{\alpha}A$ and ${}^{\alpha}B$ is computed as

$$\begin{aligned} f({}^{\alpha}A, {}^{\alpha}B) = f\big([a, b], [c, d]\big) = [l, r], \\ l = \min\big(f(a, c), f(a, d), f(b, c), f(b, d)\big), \\ r = \max\big(f(a, c), f(a, d), f(b, c), f(b, d)\big). \end{aligned} \tag{2}$$

Although any fuzzy set that satisfies the conditions of convexity and normality can be used to represent a fuzzy number, we often use triangular membership functions for their simplicity. We define a triangular fuzzy number as a 3-tuple (a, b, c), where the interval $[a, c]$ is the support and b is the peak of the fuzzy number.

2.2 Fuzzy Weighted Graphs

An environment can be represented as a fuzzy weighted graph $\widetilde{G} = (\mathcal{V}, \mathcal{E}, \mathcal{X})$, where $\mathcal{V} = (v_1, \cdots, v_N)$ is the set of vertices representing the discrete locations in the environment, $\mathcal{E}$ is the set of edges $e_k = (v_i, v_j) \in \mathcal{V} \times \mathcal{V}$ representing possible transitions from one location to another, and $\mathcal{X}$ is a set of fuzzy weights assigned to each edge. For each edge e_k, we denote a vector of fuzzy numbers $\widetilde{\boldsymbol{X}}(e_k) = \left(\widetilde{X}_1(e_k), \cdots, \widetilde{X}_r(e_k)\right)$, where each element $\widetilde{X}_i(e_k)$ represents a different measured feature of the edge e_k (e.g. length, slope, path type, etc.).

For each edge $e_k = (v_i, v_j)$, we denote $tail(e_k) = v_i$ and $head(e_k) = v_j$. An s, t path $\boldsymbol{p}$ in $\widetilde{G}$ is an n-tuple $\boldsymbol{p} = (e_1, \cdots, e_n) \in \mathcal{E}^n$ such that $head(e_i) = tail(e_{i+1})$ for $i = 1, \cdots, n-1$. We denote the start of the path as $s = tail(e_1)$, and the end of the path as $t = head(e_n)$. $\mathcal{P}(s, t)$ is the set of all s, t paths. For any path $\boldsymbol{p} \in \mathcal{P}(s, t)$, we can define an aggregated weight vector $\widetilde{\boldsymbol{F}}(\boldsymbol{p}) = \left(\widetilde{F}_1(\boldsymbol{p}), \cdots, \widetilde{F}_r(\boldsymbol{p})\right)$, where $\widetilde{F}_i(\boldsymbol{p})$ is the aggregation of all fuzzy numbers $\widetilde{X}_i(e_k), e_k \in \boldsymbol{p}$ for the feature i. The choice of aggregation function depends on the feature, as some features such as distance are well suited for a summation-type aggregation, whereas other features such as slope might be better aggregated with a maximization operator.

2.3 Bounded Rationality

An agent decision-maker trying to plan a route from point s to point t will ultimately need to choose a path from the set $\mathcal{P}(s, t)$. To do this, the agent will need to have a method for comparing paths. For a given path $\boldsymbol{p} \in \mathcal{P}(s, t)$, the aggregated weight vector $\widetilde{\boldsymbol{F}}(\boldsymbol{p})$ provides a summarization of the various measurable features of the path. Not all agents are identical, however, so we define an agent-specific interpretation $\widetilde{\boldsymbol{A}}(\boldsymbol{p}) = \left(\widetilde{A}_1(\boldsymbol{p}) = \tilde{g}_1\left(\widetilde{F}_1(\boldsymbol{p})\right), \cdots, \widetilde{A}_r(\boldsymbol{p}) = \tilde{g}_r\left(\widetilde{F}_r(\boldsymbol{p})\right)\right)$ where each function $\tilde{g}_i\left(\widetilde{F}_i(\boldsymbol{p})\right)$ is defined independently for each feature. These functions define how much various environmental properties affect the agent's interpreted cost of a path. As a rule of thumb, the values should be scaled into units corresponding to the amount of effort the agent attributes to moving along a path with each of the various features. We avoid explicitly normalizing the resulting vector $\widetilde{\boldsymbol{A}}(\boldsymbol{p})$ to allow certain features to dominate the final cost in all circumstances. For example, most agents would consider a path that contains no off-road segments to be far less costly than a path that contains several off-road segments. In this case, the off-road feature should be scaled by a very large number to guarantee that it will be the dominant factor in the final cost evaluation. Care should be taken to ensure that all of the resulting elements of $\widetilde{\boldsymbol{A}}(\boldsymbol{p})$ are all appropriately scaled.

The principle of bounded rationality states that a decision-maker cannot always consider all sources of information and tends to utilize only the most prominent features when making a decision. We implement bounded rationality using an α-level OWA operator to reduce the agent interpretation vector $\widetilde{\boldsymbol{A}}(\boldsymbol{p})$ to a single fuzzy cost value $\widetilde{C}(\boldsymbol{p})$. An α-level OWA operator is a mapping $\Phi_{\widetilde{\boldsymbol{W}}} : (\widetilde{A}_1(\boldsymbol{p}), \cdots, \widetilde{A}_r(\boldsymbol{p})) \mapsto \widetilde{C}(\boldsymbol{p})$ where $\widetilde{\boldsymbol{W}} = (\widetilde{W}_1, \cdots, \widetilde{W}_r)$ is a vector of fuzzy number weights defined on the domain $[0, 1]$. The *Alpha-Level Approach* defined in [2] provides a method to compute $\Phi_{\widetilde{\boldsymbol{W}}}$ using α-cuts. For each $\alpha \in [0, 1]$,

$$
{}^{\alpha}\Phi_{\widetilde{\boldsymbol{W}}}({}^{\alpha}\widetilde{A}_1(\boldsymbol{p}), \cdots, {}^{\alpha}\widetilde{A}_r(\boldsymbol{p})) = \left(\left. \frac{\sum_{i=1}^{r} w_i a_{\sigma(i)}}{\sum_{i=1}^{r} w_i} \right|_{\substack{w_i \in {}^{\alpha}\widetilde{W}_i \\ a_i \in {}^{\alpha}\widetilde{A}_i(\boldsymbol{p}) \\ i=1,\cdots,r}} \right), \tag{3}
$$

$$
\begin{aligned}
\text{where} \quad & \sigma : (1, \cdots, r) \to (1, \cdots, r) \\
\text{such that} \quad & a_{\sigma(i)} \geq a_{\sigma(i+1)} \ \forall \ i = 1, \cdots, r-1.
\end{aligned}
$$

From the set of ${}^{\alpha}\Phi_{\widetilde{\boldsymbol{W}}}$, the final cost value can be obtained as

$$
\widetilde{C}(\boldsymbol{p}) = \bigcup_{0 \leq \alpha \leq 1} \alpha \cdot {}^{\alpha}\Phi_{\widetilde{\boldsymbol{W}}}({}^{\alpha}\widetilde{A}_1(\boldsymbol{p}), \cdots, {}^{\alpha}\widetilde{A}_r(\boldsymbol{p})). \tag{4}
$$

An efficient algorithm to quickly compute the α-level OWA operator is given in [2]. By changing the weight vector, different aggregation operations can be defined, such as averaging the first k elements or considering only the single most influential feature.

3 Example

To demonstrate our method, consider the following hypothetical scenario. An agent is trying to reach a goal position on the opposite side of a large hill. The agent is presented with three route choices: through the woods, over the hill, or the long way around. The shortest route goes directly over the hill, however this route is very steep and unpaved. The next shortest route goes through a forrest which provides shade and has only a mild elevation change, but the route is still unpaved and also has a stream crossing with no bridge. The last route is the longest, but it is completely paved and has almost no elevation change. One can imagine three different types of agents that would prefer different routes based on their personal traits. For example, an athletic agent that does not care about elevation or path quality might prefer the direct route over the hill, whereas a less active agent might need to take the long route to avoid climbing or going off the paved route. Finally, a somewhat capable agent might prefer to take the path through the woods – even with the dirt path, water crossing, and elevation change – in order to avoid walking in the sun.

We model this scenario with the fuzzy weighted graph shown in Fig. 1. The fuzzy weights assigned to each edge are the triangular fuzzy numbers shown in

Table 1. These represent the distance, slope, path quality, amount of shade, and number of water crossings as measured by the agent. Note that the shade values are defined so that unshaded routes have a greater cost value. It is appropriate to use fuzzy numbers to represent these values, as an agent will likely not have perfect information. In this example, the route through the woods is A–B–C–E–F, the route over the hill is A–B–E–F, and the long way around is A–B–D–E–F. For each of these routes, we aggregate the features using fuzzy summation for distance, path, shade, and water, and using the fuzzy max operator for slope to represent how an agent may only care about the steepest part of a path. The resulting aggregated feature vectors are shown in Fig. 2.

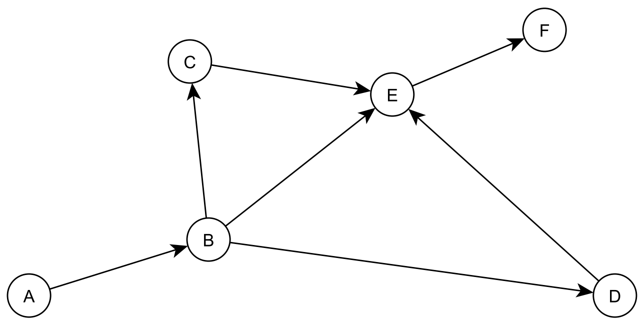

Fig. 1. Three Route Fuzzy Weighted Graph

Table 1. Fuzzy edge weights for the graph in Fig. 1

Edge	Distance	Slope	Path	Shade	Water
(A, B)	$(1, 2, 3)$	$(0, 0.64, 2.6)$	$(0, 0, 0.2)$	$(1, 2, 3)$	$(0, 0, 0.2)$
(B, C)	$(2, 4, 6)$	$(0.8, 2.8, 4.8)$	$(1.5, 3.5, 5.5)$	$(0, 0.5, 2.5)$	$(0, 0, 0.4)$
(B, D)	$(3.5, 7, 11)$	$(0, 0.57, 2.6)$	$(0, 0, 0.7)$	$(3.5, 7, 11)$	$(0, 0, 0.7)$
(B, E)	$(2.5, 5, 7.5)$	$(5.5, 7.5, 9.5)$	$(1.5, 4, 6.5)$	$(2.5, 5, 7.5)$	$(0, 0, 0.5)$
(C, E)	$(2.5, 5, 7.5)$	$(0.86, 2.9, 4.9)$	$(2, 4.5, 7)$	$(0, 0.5, 3)$	$(0, 1, 2.3)$
(D, E)	$(4, 8, 12)$	$(0, 0.7, 2.7)$	$(0, 0, 0.8)$	$(4, 8, 12)$	$(0, 0, 0.8)$
(E, F)	$(1, 2, 3)$	$(0, 0.25, 2.3)$	$(0, 0, 0.2)$	$(1, 2, 3)$	$(0, 0, 0.2)$

We now define three agent types with different interpretation functions. In this example, each feature is multiplied by a scalar value such that for an agent l, $\widetilde{A}_i^{(l)} = \tilde{g}_i^{(l)}\left(\widetilde{F}_i^{(l)}\right) = \beta_i^{(l)} \cdot \widetilde{F}_i^{(l)}$. The β values for the three agents in our example are given in Table 2. The first agent associates a moderate cost with steep and unshaded routes, as well as a high cost for water crossings. The second agent weights long routes, unshaded routes, and water crossings with an equally high cost. The third agent has a very high cost associated with steep routes, a somewhat high cost associated with unpaved routes, and a moderately high cost for water crossings.

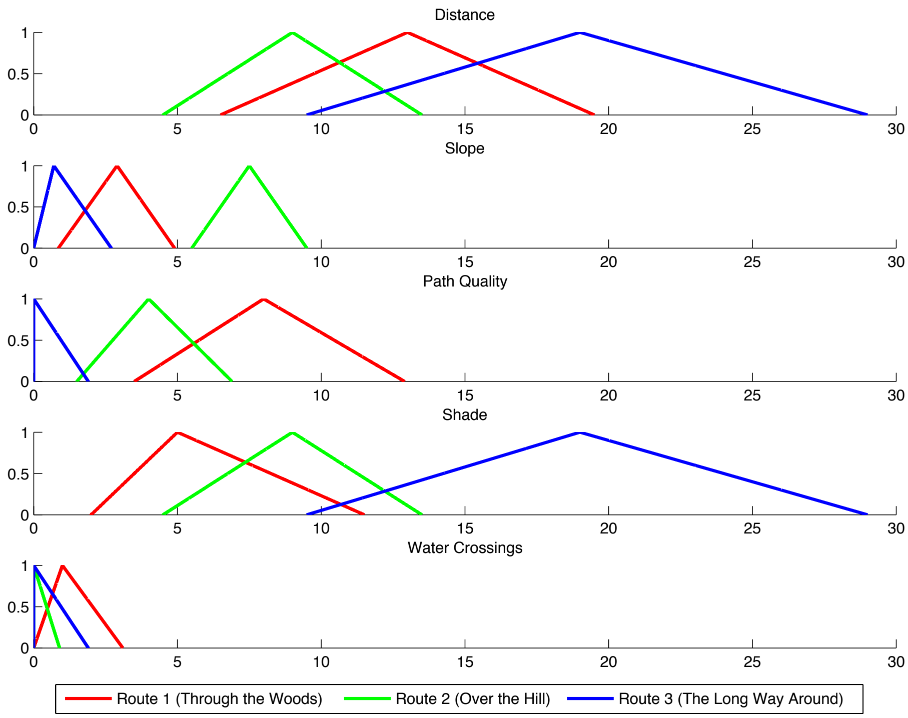

Fig. 2. Aggregation of Feature Values for Three Route Example

For each agent type, we evaluate the example environment using three different sets of fuzzy number weights, shown in Table 3. The resulting cost evaluation for each route is shown in Fig. 3. We see that the first agent tends to prefer the forrest route, but due to the large cost associated with water crossings, this agent also likes the direct route over the hill. As the OWA weights move from the single most prominent feature toward the average of all features, the distinction between routes becomes less apparent. The second agent prefers the hilly route for the "max" weights, but considers the forrest route to be about as good as the hilly route for the other weights. The third agent clearly prefers the long way around for all weight choices. From these costs, the agents can use any appropriate fuzzy ranking method to pick a route to follow. A method such as

Table 2. Agent Interpretation Values (β)

Agent	Distance	Slope	Path	Shade	Water
1	1	10	1	10	100
2	10	1	1	10	10
3	1	100	50	1	10

Table 3. α-level OWA Weights

	$\widetilde{W}_1$	$\widetilde{W}_2$	$\widetilde{W}_3$	$\widetilde{W}_4$	$\widetilde{W}_5$
$\widetilde{\boldsymbol{W}}_{(\text{Max})}$	$(0, 0.5, 1)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
$\widetilde{\boldsymbol{W}}_{(\text{Top 2})}$	$(0.5, 1, 1)$	$(0.5, 1, 1)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
$\widetilde{\boldsymbol{W}}_{(\text{Average})}$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$	$(0, 0.2, 0.4)$

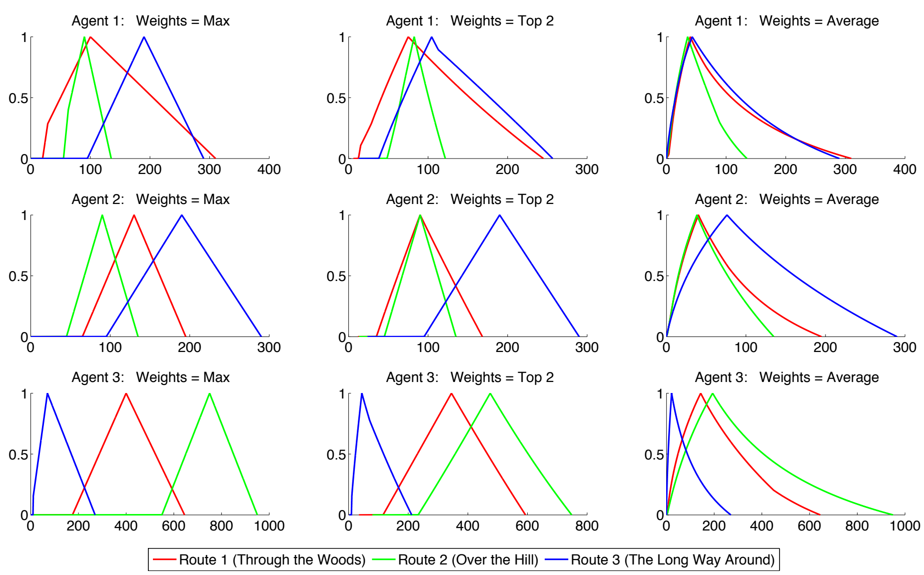

Fig. 3. Bounded Rationality Cost Evaluation for Three Route Example

the Liou and Wang index [7] that allows for an additional optimism/pessimism parameter would be appropriate for this type of problem.

4 Conclusion and Future Work

Fuzzy numbers are a natural way to represent how an agent interprets its environment. The α-level OWA operator allows an agent to aggregate multiple fuzzy route features using the principle of bounded rationality. This allows different types of agents to each interpret an environment in their own way and make unique decisions. The agent interpretation functions and the OWA weight vector are quite flexible, and can be defined to fit many different domains.

A logical extension of this work is to incorporate the cost evaluation into a general path-finding algorithm and an agent movement model. A fuzzy shortest-path algorithm that can return multiple possible routes would be a good way to provide several route choices to a decision-making agent. Agent movement could then be guided by following the least-cost route.

Acknowledgments

The authors wish to thank Dr. John Greer of the National Geospatial Intelligence Agency for support of this research under contract NGA HM 1582-10-C-0013.

References

1. Kitchin, R., Blades, M.: The Cognition of Geographic Space. I.B. Tauris, London (2002).
2. Zhou S., Chiclana F., John R.I., Garibaldi J.M.: Alpha-Level Aggregation: A Practical Approach to Type-1 OWA Operation for Aggregating Uncertain Information with Applications to Breast Cancer Treatments. IEEE Transactions on Knowledge and Data Engineering, vol. 23, no. 10, pp. 1455–1468 (2011).
3. Herbert, S.: A Behavioral Model of Rational Choice. In: Models of Man, Social and Rational: Mathematical Essays on Rational Human Behavior in a Social Setting. New York: Wiley (1957).
4. Hernandes F., Lamata M.T., Verdegay J.L., Yamakami A.: The Shortest Path Problem on Networks with Fuzzy Parameters. Fuzzy Sets and Systems, vol. 158, no. 14, pp. 1561–1570 (2007).
5. Cornelis C., De Kesel P., Kerre E.E.: Shortest Paths in Fuzzy Weighted Graphs. International Journal of Intelligent Systems, vol. 19, no. 11, pp. 1051–1068 (2004).
6. Chakraborty B., Maeda T., Chakraborty G., Multiobjective Route Selection for Car Navigation System Using Genetic Algorithm. In: Proceedings of the 2005 IEEE Midnight-Summer Workshop on Soft Computing in Industrial Applications, pp. 190–195 (2005).
7. Liou, T., Wang, M.J.: Ranking fuzzy numbers with integral value. Fuzzy Sets and Systems, vol. 50, no. 3, pp. 247–255 (1992).