Visualizing Uncertainty with Fuzzy Rose Diagrams

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Outline

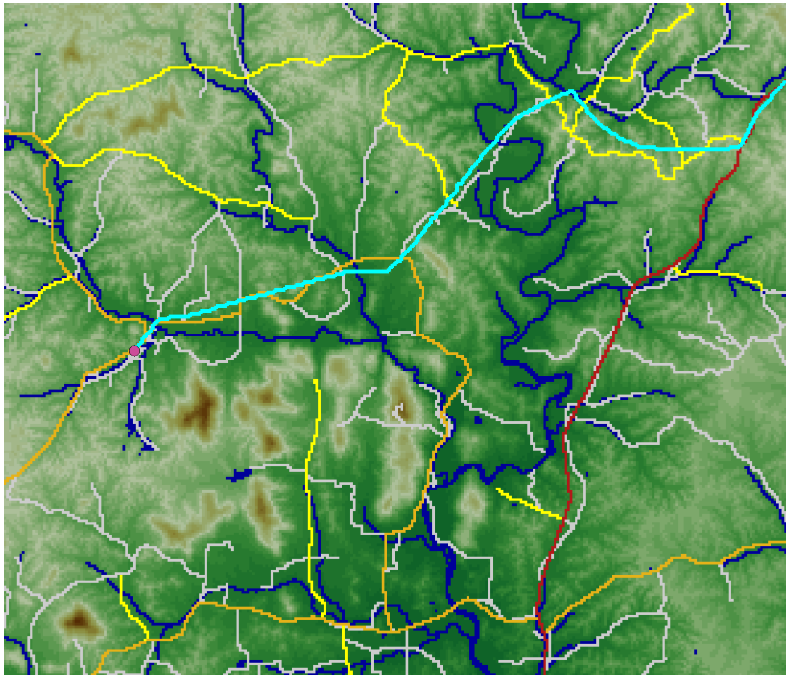
- Example Problem (Mental Maps)
- Fuzzy Numbers
- Fuzzy Rose Diagrams
  - Alpha-mapped Arcs
  - Cumulative Fuzzy Wedges
  - Cumulative Petals
- Fuzzy Weighted Graphs
- Decision Support
- Conclusion/Future Work

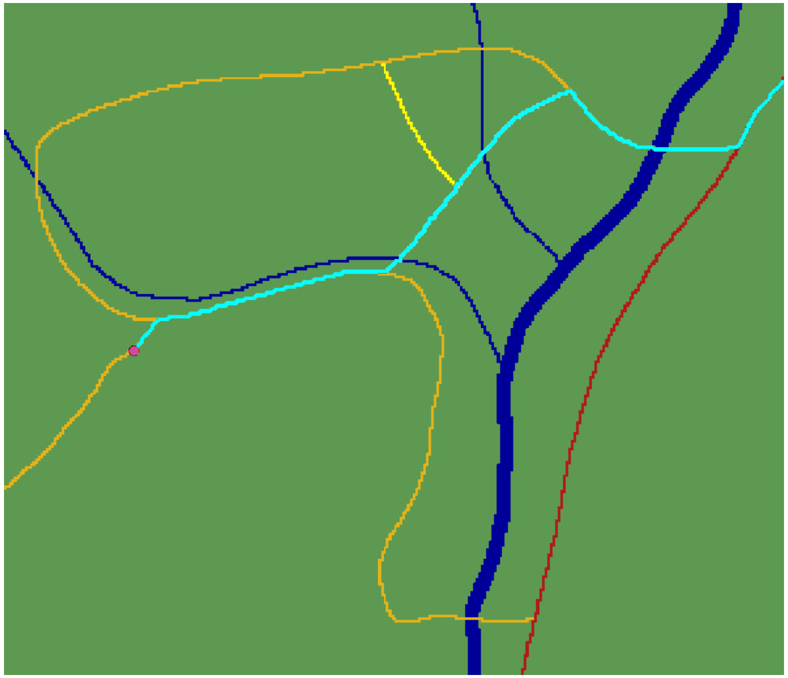
Example Problem

What does a person's mental map look like?

How does a person pick one path over another?

Is there a way to visualize the inherent uncertainty?

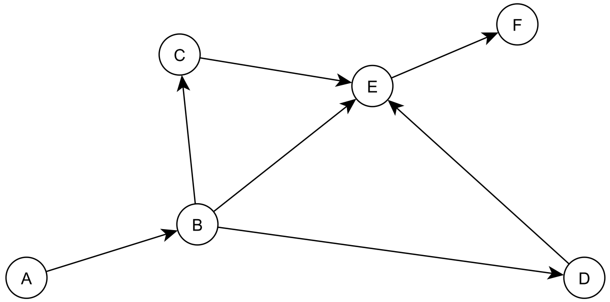




Example Problem

Example Problem

Environment is represented as a directed graph with triangular fuzzy number feature values



Edge	Distance	Slope	Path	Shade	Water
(A, B)	$(1, 2, 3)$	$(0, 0.64, 2.6)$	$(0, 0, 0.2)$	$(1, 2, 3)$	$(0, 0, 0.2)$
(B, C)	$(2, 4, 6)$	$(0.8, 2.8, 4.8)$	$(1.5, 3.5, 5.5)$	$(0, 0.5, 2.5)$	$(0, 0, 0.4)$
(B, D)	$(3.5, 7, 11)$	$(0, 0.57, 2.6)$	$(0, 0, 0.7)$	$(3.5, 7, 11)$	$(0, 0, 0.7)$
(B, E)	$(2.5, 5, 7.5)$	$(5.5, 7.5, 9.5)$	$(1.5, 4, 6.5)$	$(2.5, 5, 7.5)$	$(0, 0, 0.5)$
(C, E)	$(2.5, 5, 7.5)$	$(0.86, 2.9, 4.9)$	$(2, 4.5, 7)$	$(0, 0.5, 3)$	$(0, 1, 2.3)$
(D, E)	$(4, 8, 12)$	$(0, 0.7, 2.7)$	$(0, 0, 0.8)$	$(4, 8, 12)$	$(0, 0, 0.8)$
(E, F)	$(1, 2, 3)$	$(0, 0.25, 2.3)$	$(0, 0, 0.2)$	$(1, 2, 3)$	$(0, 0, 0.2)$

A. R. Buck, J. M. Keller, and M. Popescu, “An alpha-level OWA implementation of bounded rationality for fuzzy route selection,” in *World Conference on Soft Computing*, 2013.

Objective

How can we visualize a graph with vectors of fuzzy numbers assigned to the nodes and edges?

Goals:

- Should display all of the available information in a single graph (not a separate graph for each feature)
- Allow a high number of features
- Should be relatively easy and intuitive to interpret
- Final graphic should be reproducible across a range of mediums (i.e. print, projector, black and white)

Fuzzy Numbers

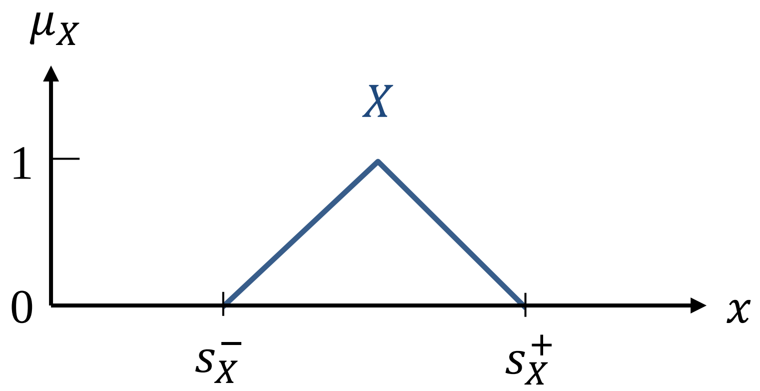
A fuzzy number is a convex, normalized fuzzy set, $X\colon \mathbb{R} \to [0, 1]$

(for this work, we only consider $\mathbb{R}^+$*)*

Defined by a membership function, $\mu_X(x)$

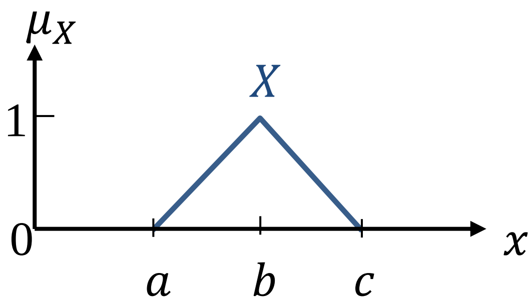
Support is given as $[s_X^-, s_X^+]$, where $\mu_X(x) > 0$

Useful for representing uncertainty such as "about 2" or "a little more than 5".

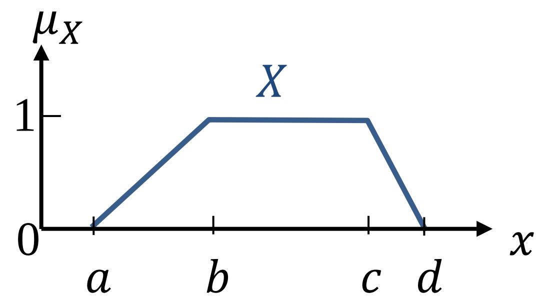


Representing Fuzzy Numbers

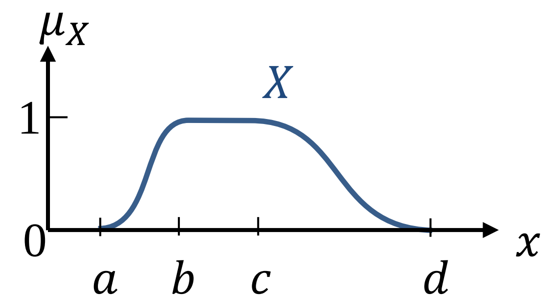
Common membership functions:



$\mu_{Tri}(x|a,b,c)$

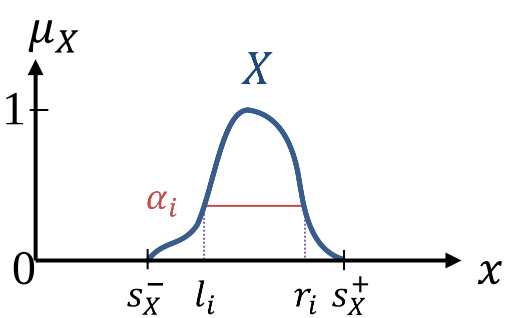


$\mu_{Trap}(x|a,b,c,d)$



$\mu_{Pi}(x|a,b,c,d)$

Alpha-cut representation:



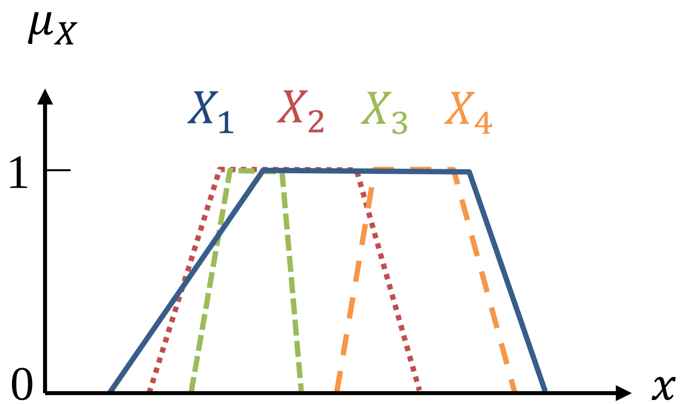
Use a fixed number of alpha-cuts

$\boldsymbol{\alpha}$	**Left**	**Right**
0	$l_0 = s_X^-$	$r_0 = s_X^+$
$\vdots$	$\vdots$	$\vdots$
α_i	l_i	r_i
$\vdots$	$\vdots$	$\vdots$
1	l_1	r_1

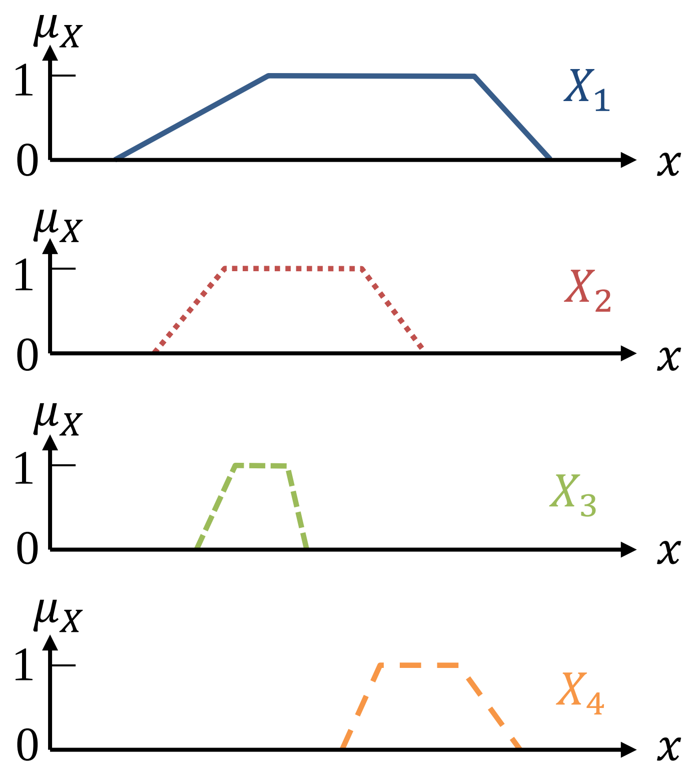
Vectors of Fuzzy Numbers

Consider visualizing a vector of fuzzy numbers, $\boldsymbol{X} = \langle X_1, \dots, X_n \rangle$

Combined Axes



Stacked Axes

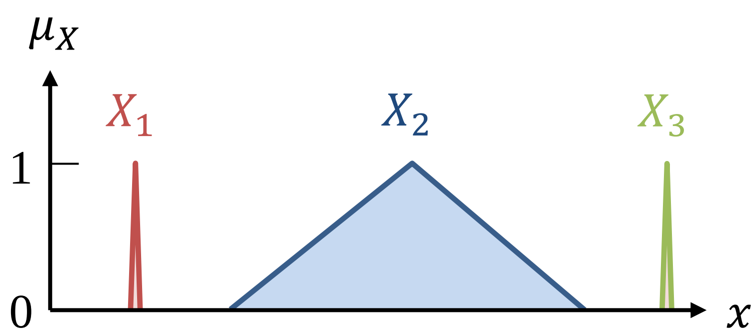


Design Principles

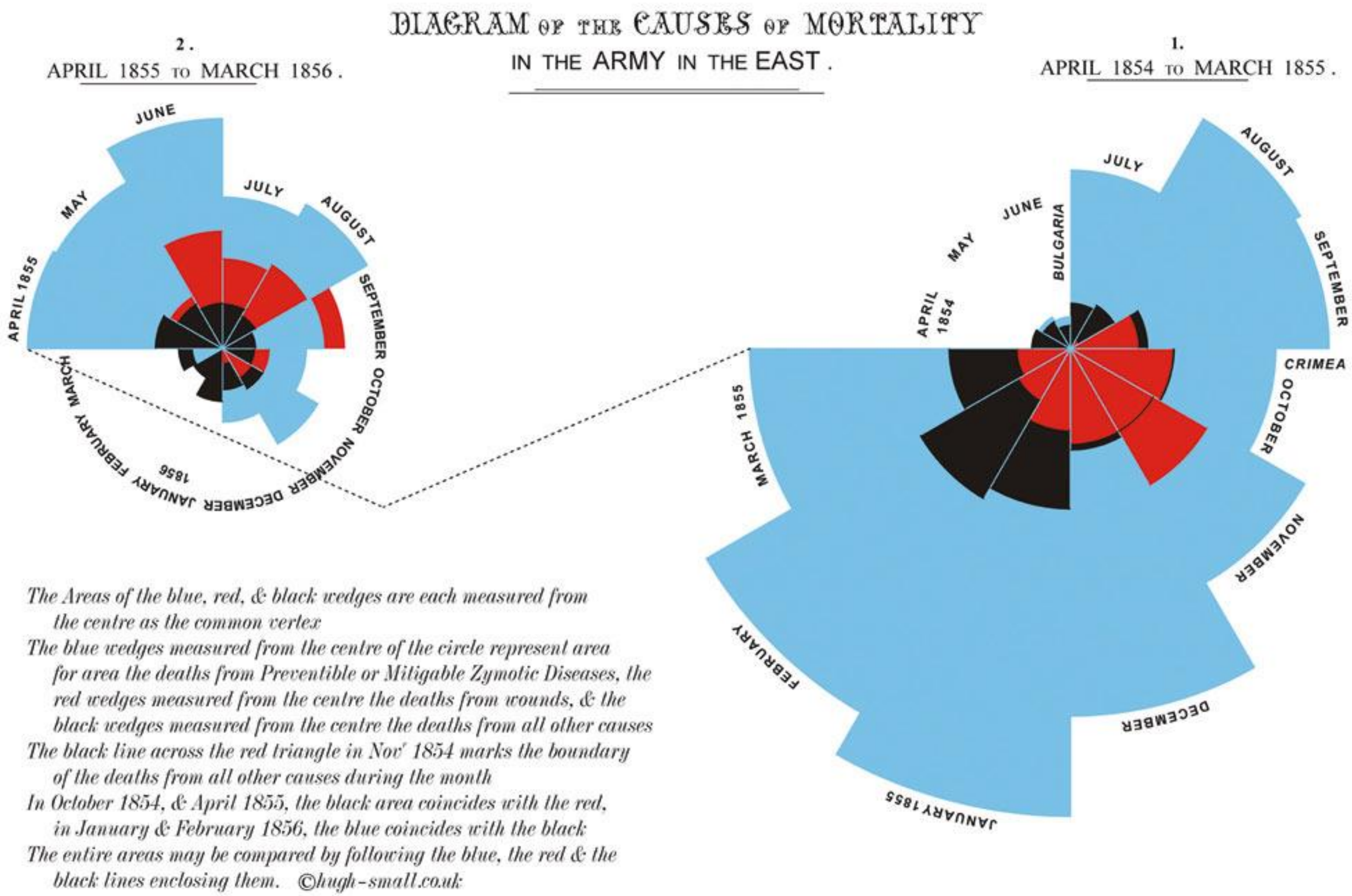
Graphics should be easy and intuitive to interpret, without misrepresenting the data.

The Principle of Perceptual Proportionality:

"The amount of ink or color used to indicate a value should be proportional to its size."

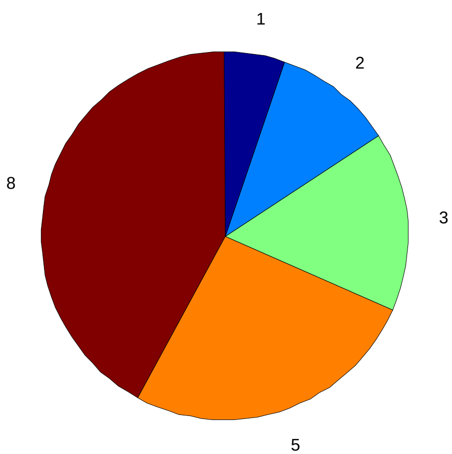


Rose Diagrams

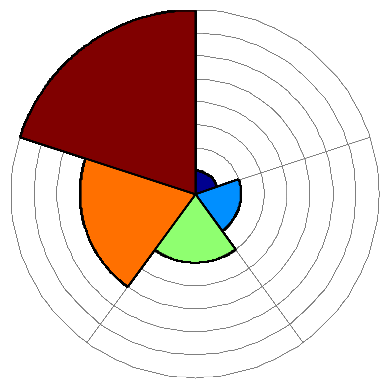


Rose Diagrams

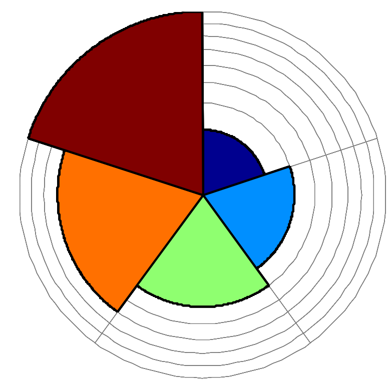
Polar area diagrams (rose diagrams, or coxcombs) are similar to traditional pie charts, but represent values by modifying the radius of each wedge instead of the angle.



Pie Chart



Proportional Radius



Proportional Area

Mapping values directly to radius length instead of area distorts the graphic.

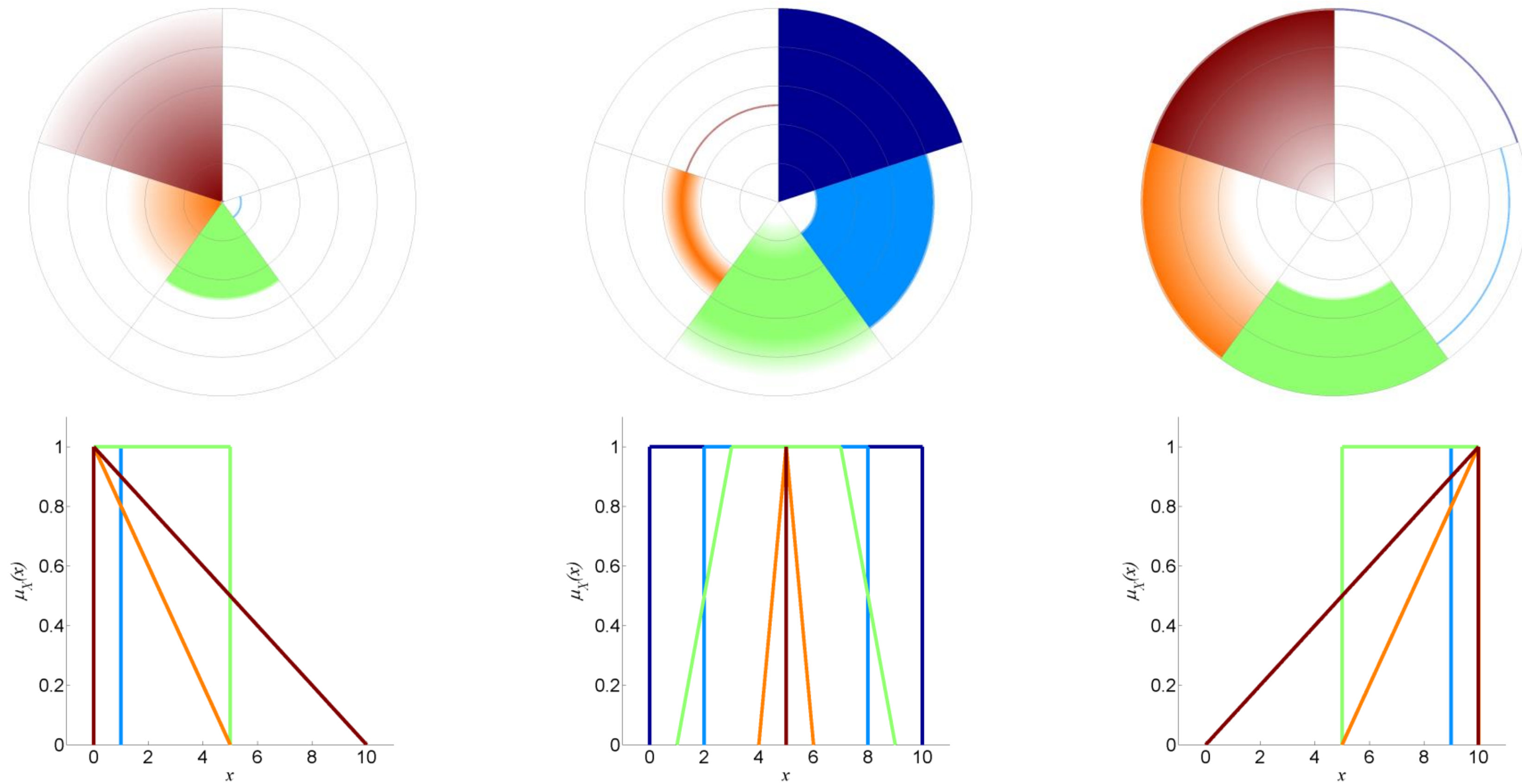
$$r_i = \sqrt{\frac{N x_i}{\pi}}$$

N: Number of features
x_i: Feature value
r_i: Radius length

Fuzzy Rose Diagrams

Alpha-mapped Arcs:

Alpha value is mapped directly to opacity.



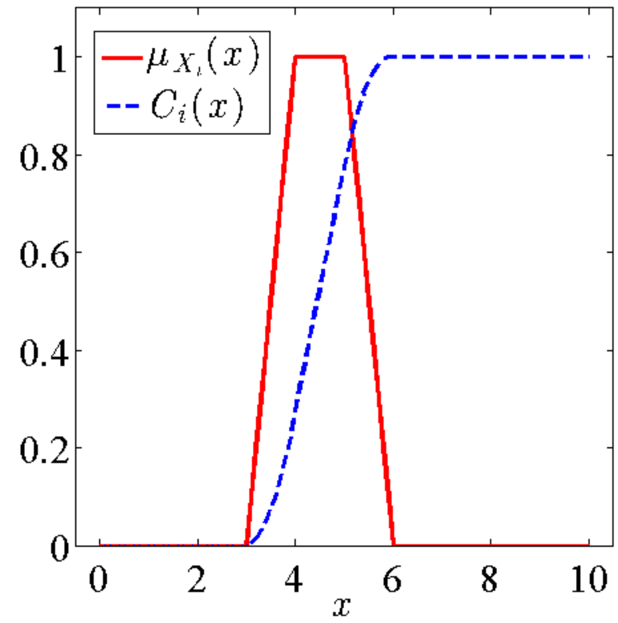
Fuzzy Rose Diagrams

The normalized cumulative membership function $C_i(x)$ of a fuzzy number X_i represents the confidence that $X_i \geq x$.

Normalized cumulative membership function:

$$C_i(x) = \frac{\int_0^x \mu_{X_i}(u)\,du}{\int_0^\infty \mu_{X_i}(u)\,du}$$

$$C_i(x) = 0 \text{ for } x \leq s_{X_i}^-$$
$$C_i(x) = 1 \text{ for } x \geq s_{X_i}^+$$



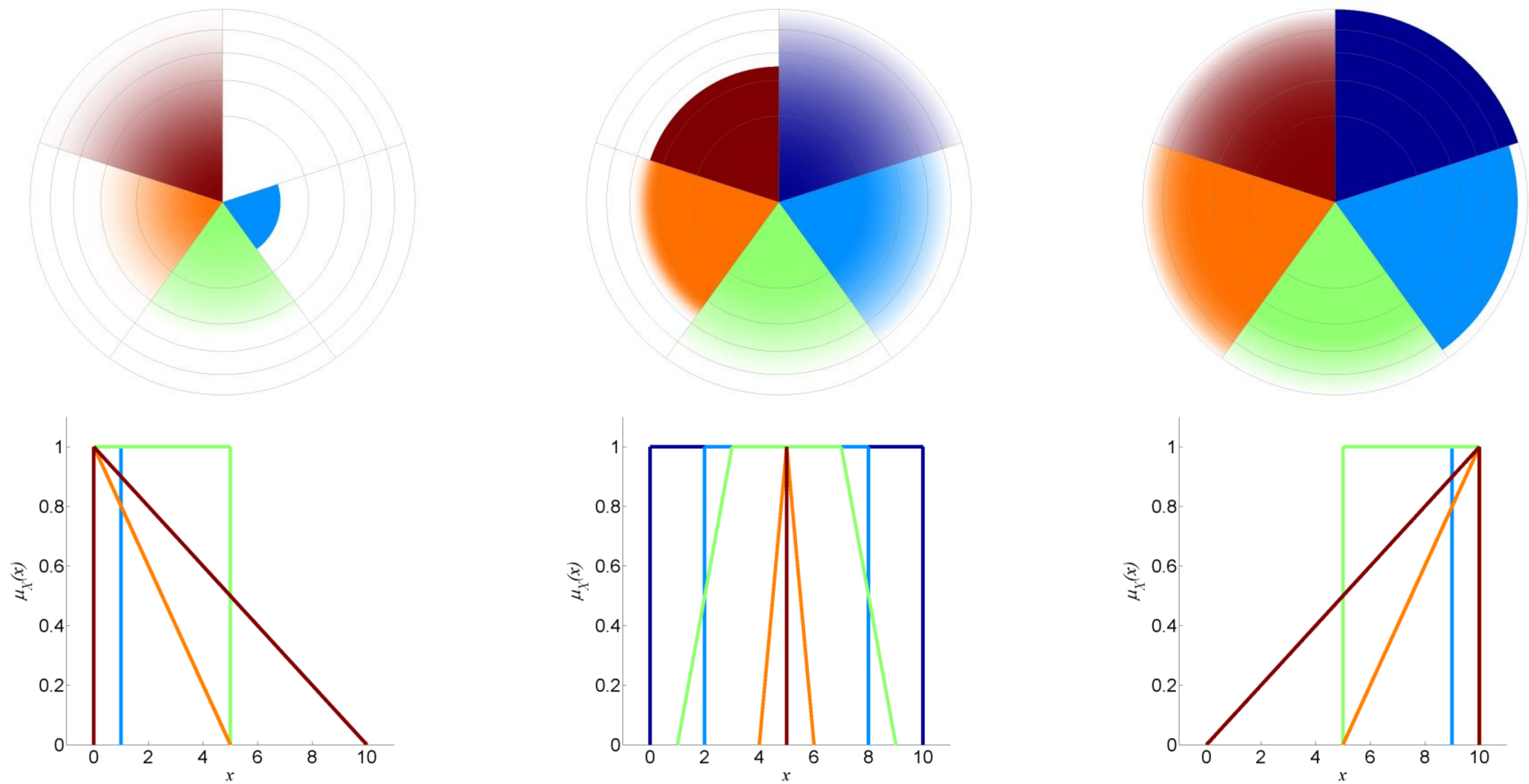
Plotted opacity value:

$$F_i(r) = 1 - C_i(x)$$
$$x = \frac{\pi r^2}{N}$$

Fuzzy Rose Diagrams

Cumulative Fuzzy Wedges:

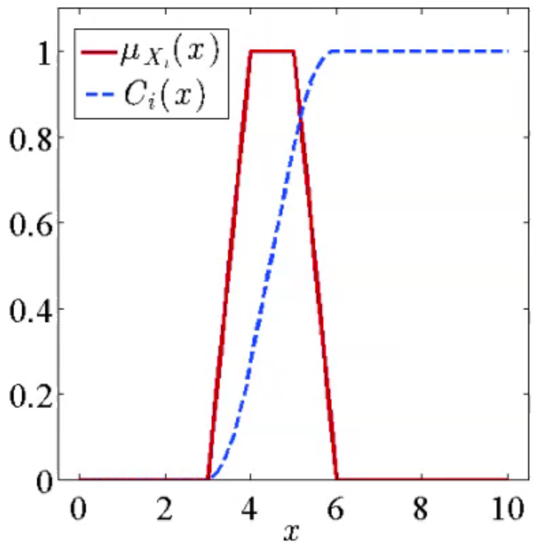
Use cumulative membership function for opacity.

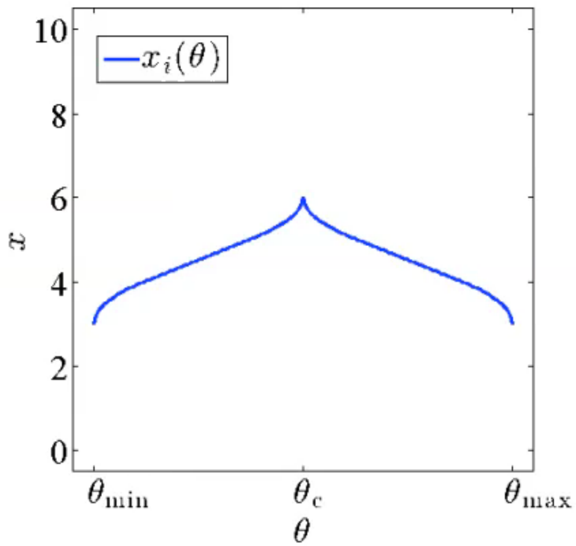


Fuzzy Rose Diagrams

Solution: Use a variable radius length for each wedge, based on the normalized cumulative membership function.

$$x_i(\theta) = C_i^{-1}\left(\frac{2|\theta - \theta_{\mathrm{c}}|}{\theta_{\max} - \theta_{\min}}\right) \qquad r_i(\theta) = \sqrt{\frac{N x_i(\theta)}{\pi}}$$

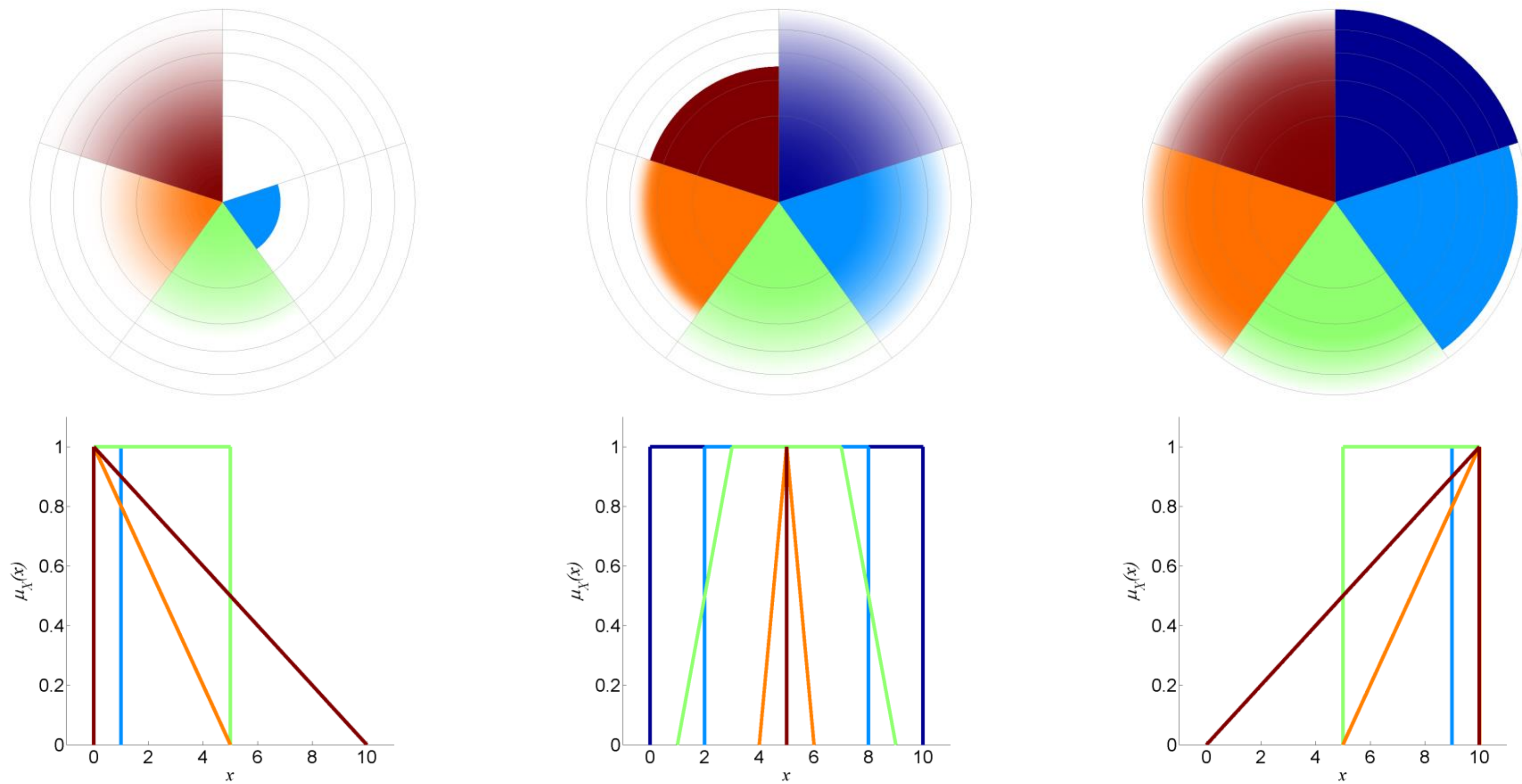




Fuzzy Rose Diagrams

Cumulative Petals:

Use cumulative membership function for radius length.
Draw min and max values with fixed opacity.

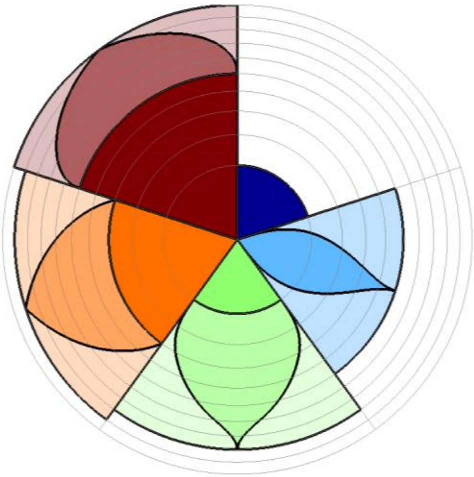


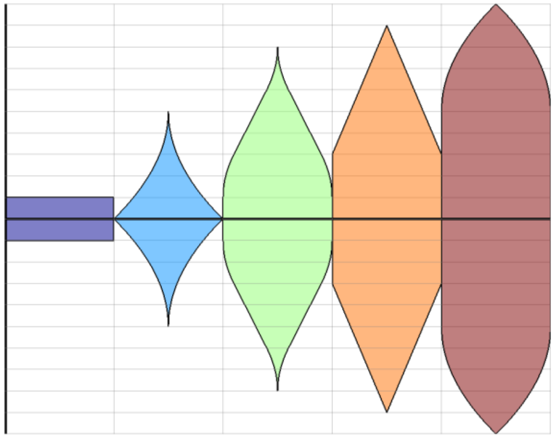
Fuzzy Weighted Graphs

How can vectors of fuzzy numbers be shown on a graph?

Solution:

Use fuzzy rose diagrams for vertices.
Unwrap the fuzzy rose diagram onto a linear axis to show edges.



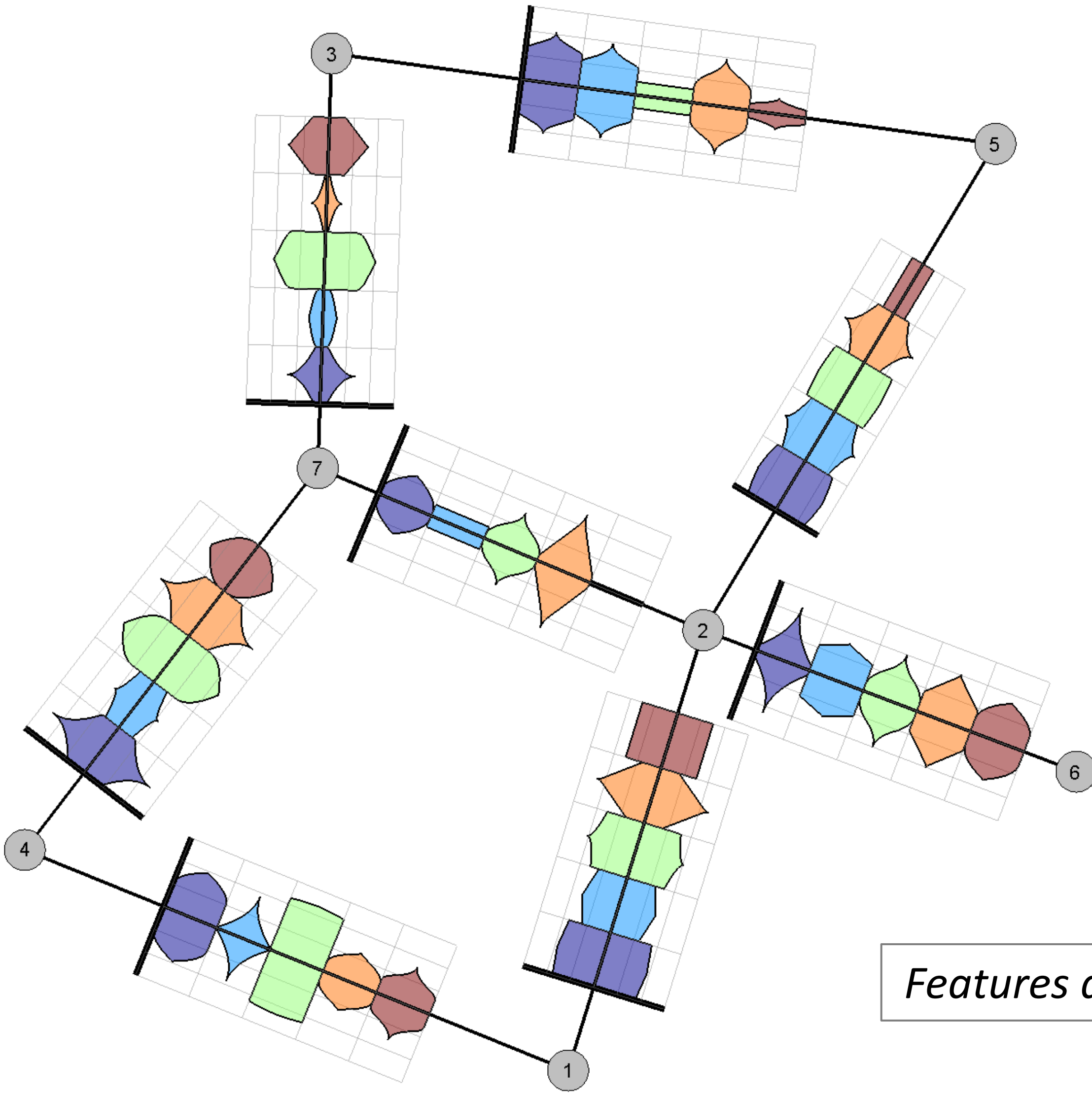


Scale factor for linear edges:

$$\gamma = \frac{\lambda^2 N}{W}$$

λ : Overall scale factor
N : Number of features
W : Width of diagram

Undirected Graphs

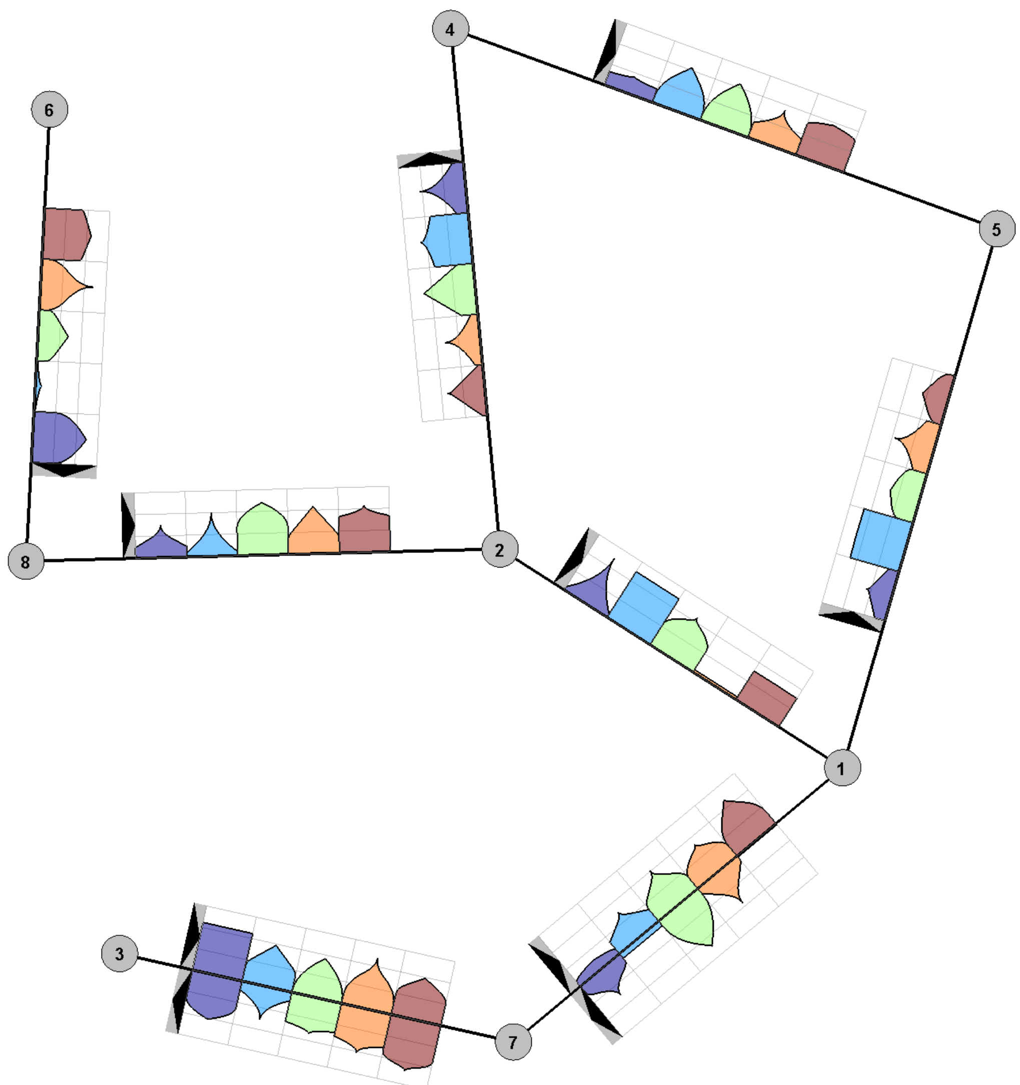


Each edge is an unwrapped linear fuzzy rose diagram.

A reference axis defines the order of the features.

Features are mirrored across the edges.

Directed Graphs



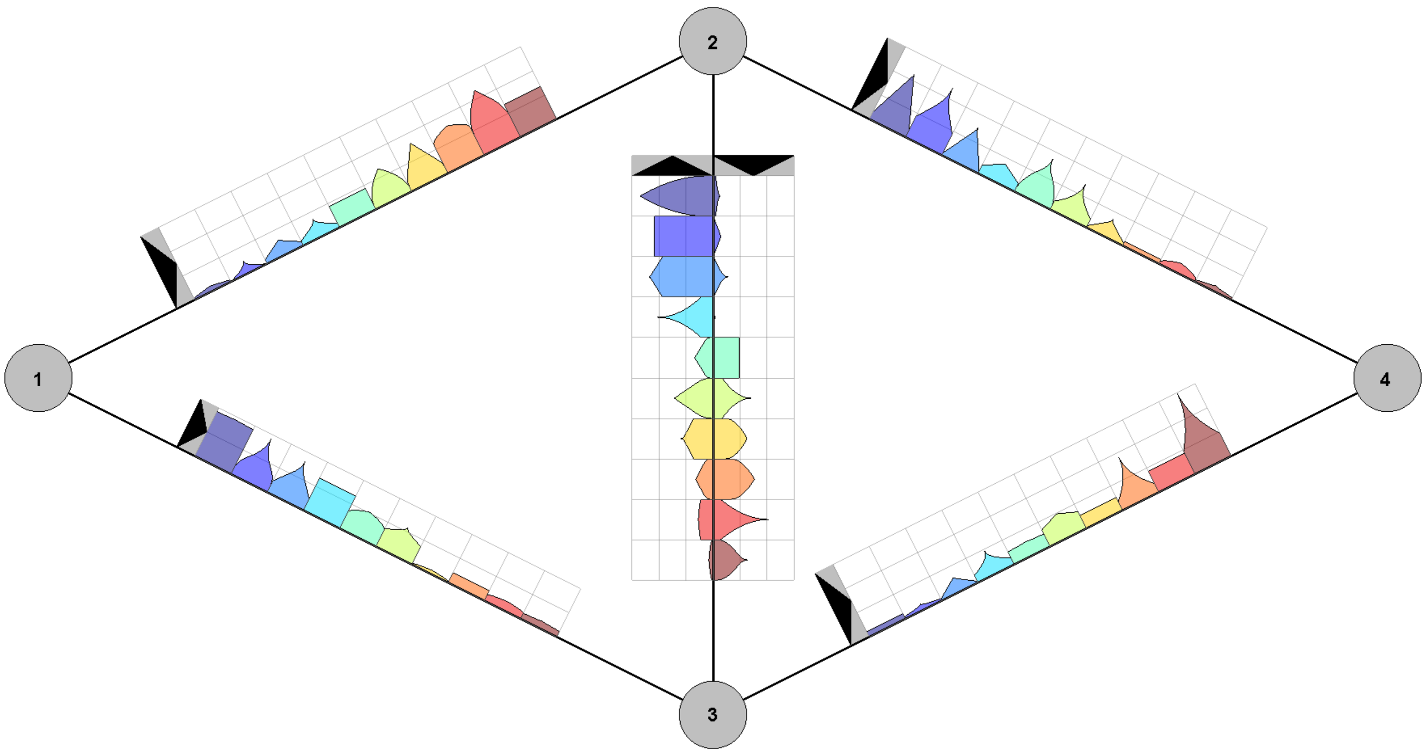
Reference axes are replaced with arrows to indicate direction.

Diagrams follow a clockwise notation.

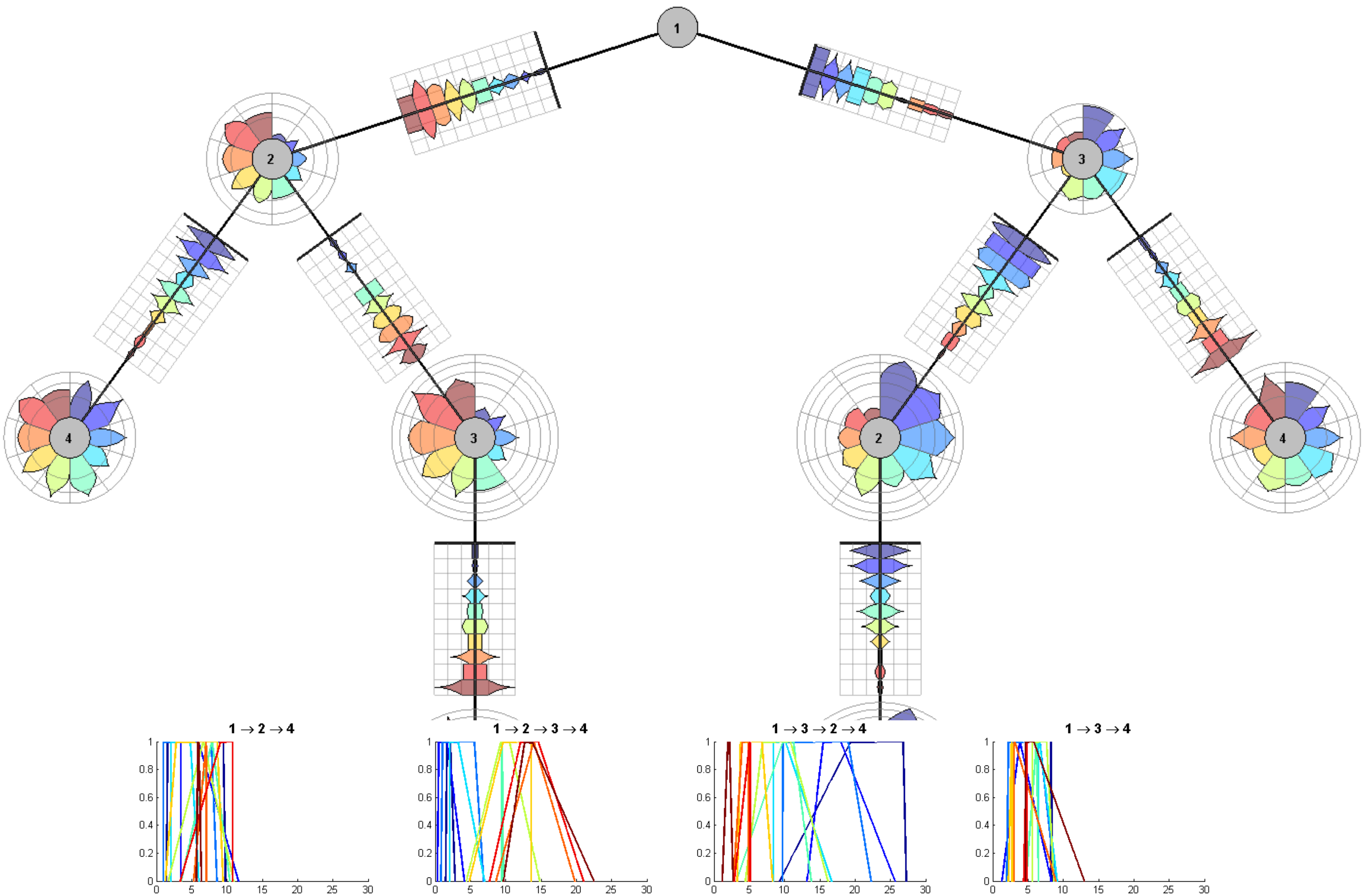
Decision Support

Consider the problem of planning a least-cost route from 1 to 4.

10 uncertain features for each edge. No obvious choice.



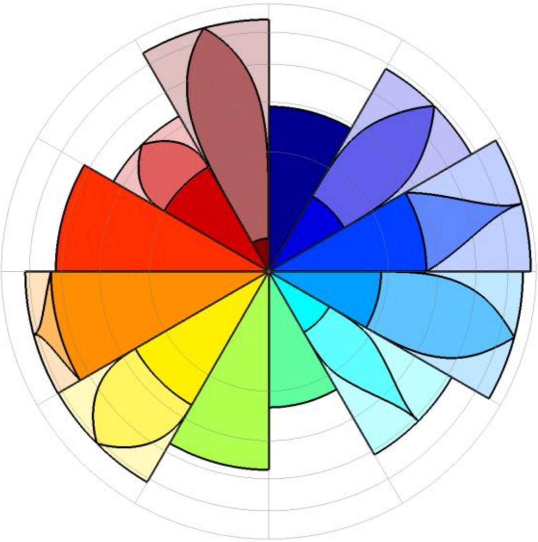
Decision Support



Conclusion

Design Goals:

- Clear, compact, and descriptive
- Principle of perceptual proportionality
- Capable of high dimensionality



Future Work:

- Validate usability with human studies
- Determine limits of usefulness (What scales are best?)
- Apply to some real world problems